

# Exercise 7: Hypothesis Testing and Summary Measures

## Compulsory e-Portfolio Component

Research Methods Module  
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## Exercise 7.1.1 & 7.1.2: Summary Measures for Diet B

### Background:

This exercise analyses weight loss data from Data Set B (Diets). The goal is to calculate summary measures for Diet B and compare them with Diet A to assess the relative effectiveness of the two weight-reducing diets.

### Excel Screenshot:

EXERCISE 7.1 - SUMMARY MEASURES WORKSHEET				
<b>Diet</b>	<b>Wtloss</b>	<b>Diet A (Example)</b>		
A	3,709		<b>Measure</b>	<b>Value</b>
A	7,087		n	50
A	6,754		Mean	5,341
A	8,994		SD	2,536
A	9,077		Median	5,642
A	6,413		Q1	3,748
A	5,877		Q3	7,033
A	2,572		IQR	3,285
A	7,52			
A	6,881	<b>Diet B (Exercises 7.1.1 &amp; 7.1.2)</b>		
A	7,265		<b>Measure</b>	<b>Value</b>
A	3,477		n	50
A	3,755		Mean	3,710
A	8,76		SD	2,769
A	7,032		Median	3,745
A	9,052		Q1	1,953
A	10,062		Q3	5,404
A	4,84		IQR	3,451
A	6,449			
A	9,019	<b>INTERPRETATION</b>		

### Results - Exercise 7.1.1 (Summary Statistics):

#### Diet B Summary Statistics:

- Sample size (n) = 50
- Sample mean weight loss = 3.710 kg
- Sample standard deviation = 2.769 kg

#### Comparison with Diet A:

- Diet A mean weight loss = 5.341 kg
- Diet A standard deviation = 2.536 kg

### Results - Exercise 7.1.2 (Quartiles):

#### Diet B Quartiles:

- Median (Q2) = 3.745 kg
- First Quartile (Q1) = 1.953 kg
- Third Quartile (Q3) = 5.404 kg

- Interquartile Range (IQR) = 3.451 kg

**Comparison with Diet A:**

- Diet A Median = 5.642 kg
- Diet A IQR = 3.285 kg

**Interpretation:**

The results clearly indicate that **Diet A is more effective than Diet B** for weight reduction:

**Central Tendency:** The mean weight loss for Diet A (5.341 kg) is approximately 1.63 kg higher than Diet B (3.710 kg). Similarly, the median for Diet A (5.642 kg) exceeds Diet B's median (3.745 kg) by about 1.9 kg. Both measures of central tendency consistently favour Diet A.

**Variability:** The standard deviations are similar (Diet A: 2.536 kg, Diet B: 2.769 kg), indicating comparable spread in weight loss outcomes for both diets. The IQRs are also similar, suggesting the middle 50% of participants experienced similarly variable results.

**Practical Significance:** Both diets produce positive average weight loss, indicating both are effective to some degree. However, Diet A produces approximately 44% more weight loss on average, which represents a clinically meaningful difference. This suggests Diet A should be recommended over Diet B for individuals seeking to maximise weight loss.

## Exercise 7.1.3: Brand Preferences Analysis for Area 2

### Background:

This exercise analyses brand preference data from Data Set D (Brandprefs). The goal is to calculate frequencies and percentage frequencies for Area 2 respondents and compare brand preference patterns between the two demographic areas.

### Excel Screenshot:

	A	B	C	D	E	F
1	<b>EXERCISE 7.1.3 - BRAND PREFERENCES FREQUENCY ANALYSIS</b>					
2						
3	<b>Area</b>	<b>Brand</b>		<b>Frequencies</b>		
4	1	B				
5	1	Other			<b>Area 1</b>	<b>Area 2</b>
6	1	A		A	11	19
7	1	B		B	17	30
8	1	Other		Other	42	41
9	1	A		<b>Total</b>	<b>70</b>	<b>90</b>
10	1	Other				
11	1	Other		<b>Percentages</b>		
12	1	Other				
13	1	Other			<b>Area 1</b>	<b>Area 2</b>
14	1	B		A	15,7	21,1
15	1	Other		B	24,3	33,3
16	1	Other		Other	60,0	45,6
17	1	A		<b>Total</b>	<b>100,0</b>	<b>100,0</b>

### Results:

#### Area 2 Brand Preferences (n = 90):

- Brand A: 19 respondents (21.1%)
- Brand B: 30 respondents (33.3%)
- Other brands: 41 respondents (45.6%)

#### Comparison with Area 1 (n = 70):

- Brand A: 11 respondents (15.7%)
- Brand B: 17 respondents (24.3%)
- Other brands: 42 respondents (60.0%)

### Interpretation:

The brand preference patterns differ notably between the two demographic areas:

**Brand A:** Shows higher preference in Area 2 (21.1%) compared to Area 1 (15.7%), a difference of 5.4 percentage points.

**Brand B:** Also shows substantially higher preference in Area 2 (33.3%) compared to Area 1 (24.3%), a difference of 9 percentage points.

**Other Brands:** The preference for competitor brands is notably lower in Area 2 (45.6%) compared to Area 1 (60.0%), a difference of 14.4 percentage points.

**Marketing Implications:** These findings suggest that the manufacturer's brands (A and B) have stronger market penetration in Area 2, with over half (54.4%) of respondents preferring either Brand A or B. In contrast, Area 1 shows weaker brand loyalty with only 40% preferring the manufacturer's brands. This could inform targeted marketing strategies—Area 1 may require more intensive promotional efforts to increase brand awareness and loyalty.

## Exercise 7.2.4: Filtration Two-Tailed Paired t-Test

### Background:

This exercise analyses filtration data from Data Set G. Each batch was split and filtered using two different agents, making a paired (related) samples t-test appropriate. The goal is to test whether the population mean impurity differs between the two filtration agents.

### Excel Screenshot:

	A	B	C	D	E	F	G	H	I
1	<b>EXERCISES 7.2.2 &amp; 7.2.4 - FILTRATION DATA HYPOTHESIS TESTING</b>								
2	<i>Related (Paired) Samples t-Test</i>								
3									
4	<b>Batch</b>	<b>Agent1</b>	<b>Agent2</b>	<b>Difference</b>		<b>Summary Statistics</b>			
5	1	7,7	8,5	-0,8		<b>Statistic</b>	<b>Agent 1</b>	<b>Agent 2</b>	
6	2	9,2	9,6	-0,4		Mean	8,250	8,683	
7	3	6,8	6,4	0,4		Variance	1,059	1,078	
8	4	9,5	9,8	-0,3		n	12	12	
9	5	8,7	9,3	-0,6					
10	6	6,9	7,6	-0,7		<b>Exercise 7.2.4: Paired t-test (Two-tailed)</b>			
11	7	7,5	8,2	-0,7		Mean Difference	-0,433		
12	8	7,1	7,7	-0,6		SD of Differences	0,460		
13	9	8,7	9,4	-0,7		SE of Mean Diff	0,133		
14	10	9,4	8,9	0,5		t Statistic	-3,264		
15	11	9,4	9,7	-0,3		df	11		
16	12	8,1	9,1	-1		p-value (two-tailed)	0,0075		
17									
18						<b>Exercise 7.2.2: One-tailed Test</b>			
19						p-value (one-tailed)	0,0038		
20									

### Hypotheses:

$$H_0: \mu_1 = \mu_2 \text{ (no difference in mean impurity between agents)}$$

$$H_1: \mu_1 \neq \mu_2 \text{ (mean impurity differs between agents)}$$

### Results:

#### Summary Statistics:

- Agent 1 mean impurity: 8.250 parts per 1000
- Agent 2 mean impurity: 8.683 parts per 1000
- Mean difference: -0.433 parts per 1000
- SD of differences: 0.460

#### Test Statistics:

- t statistic: -3.264
- Degrees of freedom: 11
- p-value (two-tailed): 0.0075

### Interpretation:

**Statistical Conclusion:** The two-tailed p-value (0.0075) is less than 0.05, so we **reject the null hypothesis** at the 5% significance level. There is significant evidence that the population mean impurity differs between the two filtration agents.

**Practical Interpretation:** Agent 1 produces a mean impurity of 8.250 parts per 1000, which is 0.433 parts per 1000 lower than Agent 2's mean of 8.683. This difference is statistically significant ( $p = 0.0075$ , which is significant at the 1% level).

**Recommendation:** Since lower impurity indicates better filtration performance, Agent 1 appears to be the more effective filtration agent and should be preferred for applications where minimising impurity is important.

## Exercise 7.2.2: Filtration One-Tailed Test

### Background:

Building on Exercise 7.2.4, we now conduct a one-tailed test to determine whether Filter Agent 1 is specifically more effective (produces lower impurity) than Agent 2.

### Hypotheses:

$H_0: \mu_1 \geq \mu_2$  (Agent 1 impurity is greater than or equal to Agent 2)  
 $H_1: \mu_1 < \mu_2$  (Agent 1 is more effective—produces lower impurity)

### Results:

#### Consistency Check:

The sample data show Agent 1 mean (8.250) < Agent 2 mean (8.683).  
The data are consistent with  $H_1$  (Agent 1 more effective).

#### Test Statistics:

- t statistic: -3.264
- Degrees of freedom: 11
- p-value (one-tailed): 0.0038

### Interpretation:

**Statistical Conclusion:** The one-tailed p-value (0.0038) is less than 0.01, so we **reject the null hypothesis** at the 1% significance level. There is strong evidence that Filter Agent 1 is more effective at removing impurities than Agent 2.

**Comparison with Two-Tailed Test:** The one-tailed test provides stronger evidence ( $p = 0.0038$  vs  $p = 0.0075$ ) because we had a directional hypothesis. The one-tailed p-value is exactly half the two-tailed p-value when the sample mean difference is in the predicted direction.

**Conclusion:** There is statistically significant evidence at the 1% level that Filter Agent 1 produces lower impurity levels, making it the preferred choice for filtration applications.

## Exercises 7.2.3 & 7.2.5: Bank Cardholder Income Analysis

### Background:

This exercise analyses bank cardholder data from Data Set C (Superplus). The goal is to test whether the population mean income for male cardholders exceeds that of female cardholders. Since the male and female samples are independent, an independent samples t-test is appropriate.

### Excel Screenshot:

EXERCISES 7.2.3 & 7.2.5 - BANK CARDHOLDER INCOME ANALYSIS							
Independent Samples t-Test							
	Sex	Income	Male Income	Female Income	Statistic	Males	Females
5	M	40,6	40,6	33,1	Mean	52,913	44,233
6	M	54,6	54,6	35,8	Variance	233,129	190,176
7	M	38,6	38,6	68,8	SD	15,269	13,790
8	M	58,2	58,2	31,6	n	60	60
9	M	34,6	34,6	38,2	<b>F-test for Equality of Variances</b>		
10	M	42,9	42,9	42	F statistic	1,226	
11	M	67,5	67,5	33,4	df1, df2	59	59
12	M	79,8	79,8	50,3	p-value (two-tail)	0,4365	
13	M	54,4	54,4	39,6	<b>Independent t-test (One-tailed: Males &gt; Females)</b>		
14	M	47,3	47,3	30,7	Difference in Means	8,680	
15	M	66,4	66,4	31,3	Pooled Variance	211,652	
16	M	69	69	61,3	SE of Difference	2,656	
17	M	62	62	30	t Statistic	3,268	
18	M	52,5	52,5	38,1	df	118	
19	M	72,6	72,6	56,4	p-value (two-tailed)	0,0014	
20	M	52,4	52,4	35,7	p-value (one-tailed)	0,0007	
21	M	59,5	59,5	31,3			
22	M	59,1	59,1	40,4			
23	M	36,7	36,7	32,1			
24	M	54,6	54,6	66,4			

### Step 1: F-test for Equality of Variances

#### Hypotheses:

$H_0: \sigma^2_{\text{males}} = \sigma^2_{\text{females}}$  (variances are equal)

$H_1: \sigma^2_{\text{males}} \neq \sigma^2_{\text{females}}$  (variances are unequal)

#### Results:

- Male variance: 233.129
- Female variance: 190.176
- F statistic: 1.226
- df: (59, 59)
- p-value (two-tailed): approximately 0.44

**Conclusion:**  $p > 0.05$ , so we fail to reject  $H_0$ . We assume equal variances and use the equal variances form of the independent samples t-test.

### Step 2: Independent Samples t-Test (One-tailed)

### Hypotheses:

$H_0: \mu_{\text{males}} \leq \mu_{\text{females}}$  (male mean income does not exceed female)

$H_1: \mu_{\text{males}} > \mu_{\text{females}}$  (male mean income exceeds female)

### Summary Statistics:

- Male mean income: £52.913k (n = 60)
- Female mean income: £44.233k (n = 60)
- Difference in means: £8.680k
- Pooled variance: 211.652

### Test Statistics:

- t statistic: 3.268
- Degrees of freedom: 118
- p-value (one-tailed): 0.0007

### Interpretation:

**Statistical Conclusion:** The one-tailed p-value (0.0007) is less than 0.01, so we **reject the null hypothesis** at the 1% significance level. There is strong evidence that the population mean income for male Superplus Diamond cardholders exceeds that of female cardholders.

**Practical Interpretation:** Male cardholders have a mean income of £52,913, which is £8,680 higher than the female mean income of £44,233. This represents a 19.6% higher income for males on average.

### Assumptions and Validation:

1. **Independence:** The samples are independently drawn—each cardholder appears in only one group.
2. **Normality:** Income should be approximately normally distributed. With n=60 in each group, the Central Limit Theorem supports the validity of the t-test even if distributions are slightly non-normal. Validation could be performed using normal probability plots (Q-Q plots) or histograms for each group.
3. **Equal Variances:** The F-test ( $p \approx 0.44$ ) confirms this assumption is reasonable.
4. **Random Sampling:** We assume cardholders were randomly selected from the Superplus Diamond cardholder population.